NUMERICKÁ INTEGRACE

- neznáme explicitní tvar fce } přibližný výpočet integrálu
- známe ho, ale je složitý }

→ metody jsou založené na nahrazení fce interpolačním polynomem →

Obecná formulace úlohy

$$\int_a^b f(x)\,dx \qquad a<b \in R$$

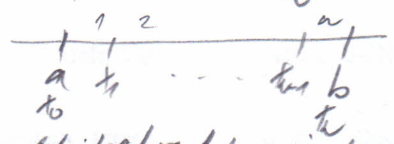
→ Newton-Cotesovy kvadraturní vzorce – uzavřeného typu – uzly → krajní body
– otevřeného typu – uzly → středy

NCKV uzavřeného typu

$\langle a,b\rangle$ rozdělen na ekvidistantní uzly

$$a = x_0 < x_1 < \dots < x_n = b$$

$h = \frac{b-a}{n}$



→ na každém intervalu $\langle x_{i-1}, x_i\rangle$, $i=1\dots n$ nahradíme integrand LIP stupně k $L_{i,k}$:

$$\int_{x_{i-1}}^{x_i} f(x)\,dx \approx \int_{x_{i-1}}^{x_i} L_{i,k}(x)\,dx$$

⇒ jednoduchý NCKV stupně k

→ pak na celém intervalu $\langle a,b\rangle$ platí složený NCKV

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f(x)\,dx \approx \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} L_{i,k}(x)\,dx$$

→ Stanovení chyby $R_k(f)$

a) na $\langle x_{i-1}, x_i\rangle$

$$R_{i,k}(f) = \int_{x_{i-1}}^{x_i} f(x)\,dx - \int_{x_{i-1}}^{x_i} L_{i,k}(x)\,dx =$$

$$= \int_{x_{i-1}}^{x_i} \frac{f^{(k+1)}(\eta_i)}{(k+1)!}(x-t_0)\dots(x-t_k)\,dx$$

t_k … vnitřní uzly ke konstrukci LIP

$f^{(k+1)}(\eta_i)$ odhadujeme $M = \max_{x\in\langle a,b\rangle} |f^{(k)}(x)|$

b) na $\langle a,b\rangle$

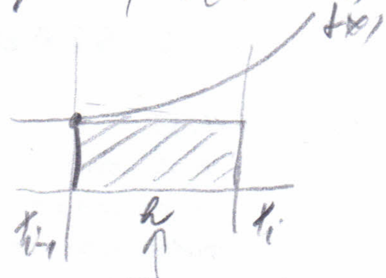
$$R_{n,k}(f) = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \frac{f^{(k+1)}(\eta_i)}{(k+1)!}(x-t_0)\dots(x-t_k)\,dx$$

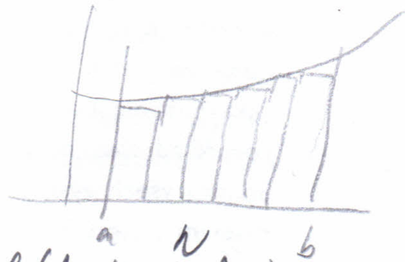
OBDÉLNÍKOVÁ METODA (k=0) NCKV

$f(x) \approx$ na každém podintervalu polynomem 0-tého stupně (konstantou)

na $\langle x_{i-1}; x_i \rangle$ $\quad f(x) \approx L_{1,0} = f(x_{i-1})$

$$\int_{x_{i-1}}^{x_i} f(x)\,dx \approx \int_{x_{i-1}}^{x_i} f(x_{i-1})\,dx = f(x_{i-1}) \cdot [x]_{x_{i-1}}^{x_i} = f(x_{i-1})(x_i - x_{i-1})$$





na $\langle a; b \rangle$

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} L_{1,0}(x)\,dx = \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} f(x_{i-1})\,dx = h \sum_{i=1}^{N} f(x_{i-1}) = h(f_0 + f_1 + \dots f_{N-1})$$

↳ složené obd. pravidlo

CHYBA:

$$R_0(f) = \int_a^b f(x)\,dx - h \sum_{i=1}^{N} f(x_{i-1}) = \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} \frac{f'(\eta)}{1!}(x - x_{i-1})\,dx =$$

$$= \left| \begin{array}{l} \text{substituce} \quad h = \frac{b-a}{N} \\ x = x_{i-1} + th, \; t \in \langle 0, 1 \rangle \\ dx = h\,dt \end{array} \right| = \sum_{i=1}^{N} \frac{f'(\eta)}{1!} \int_{x_{i-1}}^{x_i} (x - x_{i-1})\,dx = \sum_{i=1}^{N} M_i \frac{h^2}{2}$$

$$\int_{x_{i-1}}^{x_i} (x - x_{i-1})\,dx = \int_0^1 th \cdot h\,dt = h^2 \int_0^1 t\,dt = h^2 \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2} h^2$$

$x - x_{i-1} = th$

$dx = h\,dt$

pro $x = x_{i-1}$: $0 = th \rightarrow t = 0$

$x = x_i$: $\underbrace{x_i - x_{i-1}}_{h} = th$

$h = th$

$h - th = 0$

$h(1-t) = 0 \quad t = 1$

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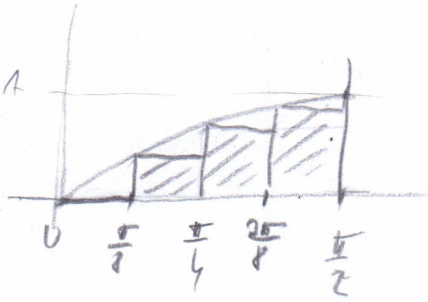
na $\langle a; b \rangle$ $\quad |R_0(f)| \le M_1 \cdot \frac{h^2}{2} \cdot N = M_1 \cdot \frac{(b-a)^2}{2N}$ $\qquad M_1 = \max_{x \in \langle a; b \rangle} |f'(x)|$

Př: $\int_0^{\frac{\pi}{2}} \sin x \, dx$ { přesná hodnota: $[-\cos x]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} + \cos 0 = 0 + 1 = \underline{\underline{1}}$

x	sin x
0	0
$\frac{\pi}{8}$	0,3827
$\pi/4$	0,7071
$3\pi/8$	0,9239
$\pi/2$	1

$$\int_0^{\frac{\pi}{2}} \sin x \doteq \frac{\pi}{8}\left(0 + 0{,}3827 + 0{,}7071 + 0{,}9239\right) =$$
$$\doteq 0{,}7884$$



$n = 4 \quad h = \frac{\pi}{8}$

Odhad chyby: $M_1 \cdot \frac{\left(\frac{\pi}{2}-0\right)^2}{2\cdot 4} = 1\,\frac{\left(\frac{\pi}{2}\right)^2}{8} = \frac{\pi^2}{4}\cdot\frac{1}{8} = \frac{\pi^2}{32} = 0{,}3084$

$f(x) = \sin x$

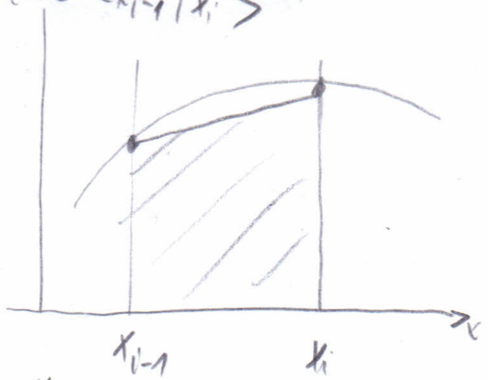
$f'(x) = \cos x \rightarrow \max$ na $\langle 0; \frac{\pi}{2}\rangle$ je 1

[$\Rightarrow$ přesná hodnota leží v $(0{,}7884 \pm 0{,}3084)$]

LICHOBĚŽNÍKOVÁ METODA (NCKV 1. stupně)

- nahrazujeme v každém $\langle x_{i-1}; x_i\rangle$ $i = 1 \dots N$ $f(x)$ $L_{i,1}(x) = f(x_{i-1})\frac{x-x_i}{x_{i-1}-x_i} + f(x_i)\frac{x-x_{i-1}}{x_i-x_{i-1}}$; $i = 1 \dots N$

na $x \in \langle x_{i-1}; x_i\rangle$



$L_{i,1}(x)$; $h = x_i - x_{i-1}$

$$\left.\begin{matrix}[x_{i-1};\, f(x_{i-1})] \\ [x_i;\, f(x_i)]\end{matrix}\right\} L_{i,1}(x) = f(x_{i-1})\cdot\frac{x-x_i}{x_{i-1}-x_i} + f(x_i)\frac{x-x_{i-1}}{x_i-x_{i-1}}$$

$$\int_{x_{i-1}}^{x_i} f(x)\,dx \doteq \int_{x_{i-1}}^{x_i}\left[f(x_{i-1})\frac{x-x_i}{x_{i-1}-x_i} + f(x_i)\frac{x-x_{i-1}}{x_i-x_{i-1}}\right]dx = \frac{f(x_{i-1})}{\underbrace{x_{i-1}-x_i}_{-h}}\int_{x_{i-1}}^{x_i}(x-x_i)\,dx + \frac{f(x_i)}{\underbrace{x_i-x_{i-1}}_{h}}\int_{x_{i-1}}^{x_i}(x-x_{i-1})\,dx$$

$=$ | subst: $x = x_{i-1} + th$; $t \in \langle 0; 1\rangle$
$dx = h\,dt$
pro $x = x_{i-1}$: $\cancel{x_{i-1}} = \cancel{x_{i-1}} + th \Rightarrow t = 0$
$x = x_i$: $x_i = x_{i-1} + th \quad h(1-t) = 0 \Rightarrow t = 0$
$x_i - x_{i-1} = th$
$h = th$ |

$=$ | pro chybu: $\varnothing$
$x - x_{i-1} = \cancel{x_{i-1}} + th - \cancel{x_{i-1}}$
$x - x_i = x_{i-1} + th - x_i = th - h$
$= h(t-1)$ |

$$= -\frac{f(x_{i-1})}{h}\int_0^1 (x_{i-1}+th-x_i)h\,dt + \frac{f(x_i)}{h}\int_0^1 (\cancel{x_{i-1}}+th-\cancel{x_{i-1}})h\,dt =$$

$$th-\underbrace{(x_i-x_{i-1})}_{h} = h(t-1)$$

$$= -\frac{f(x_{i-1})}{\cancel{h}}\cdot h^{\cancel{2}}\int_0^1 (t-1)\,dt + \frac{f(x_i)}{\cancel{h}}h^{\cancel{2}}\int_0^1 t\,dt = -h f(x_{i-1})\left[\frac{t^2}{2}-t\right]_0^1 + h f(x_i)\left[\frac{t^2}{2}\right]_0^1 =$$

$$= -h f(x_{i-1})\Big[\underbrace{\left(\tfrac{1}{2}-1\right)-(0-0)}_{-\frac{1}{2}}\Big] + h f(x_i)\Big[\underbrace{\tfrac{1}{2}-\tfrac{0}{2}}_{\frac{1}{2}}\Big] = \frac{h}{2}f(x_{i-1}) + \frac{h}{2}f(x_i) = \frac{h}{2}[f(x_{i-1})+f(x_i)]$$

na $\langle a;b\rangle$

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N}\frac{h}{2}[f(x_{i-1})+f(x_i)] = \frac{h}{2}\Big\{\underbrace{[f(x_0)+f(x_1)]}_{i=1} + \underbrace{[f(x_1)+f(x_2)]}_{i=2} + \dots + \underbrace{[f(x_{N-2})+f(x_{N-1})]}_{i=N-1} +$$

$$+ \underbrace{[f(x_{N-1})+f(x_N)]}_{i=N}\Big\} = \frac{h}{2}\Big[\underset{f(a)}{f(x_0)} + 2\sum_{i=2}^{N-1} f(x_i) + \underset{f(b)}{f(x_N)}\Big]$$

Odhad chyby: $$R_{1,i}(f) = \frac{f''(\eta)}{2!}\int_{x_{i-1}}^{x_i}(x-x_{i-1})(x-x_i)\,dx = |\text{stejná substituce}| =$$

$$= \frac{f''(\eta)}{2}\int_0^1 th\cdot h(t-1)h\,dt = \frac{f''(\eta)}{2}\cdot h^3\int_0^1 (t^2-t)\,dt = \frac{f''(\eta)}{2}h^3\Big[\underbrace{\frac{t^3}{3}-\frac{t^2}{2}}_{\frac{1}{3}-\frac{1}{2}=-\frac{1}{6}}\Big]_0^1 = -\frac{1}{12}h^3 \underset{M_2}{f''(\eta)}$$

$$|R_1(f)| \le N\cdot\frac{1}{12}h^3 M_2 = \frac{N\cdot M_2}{12}\frac{(b-a)^3}{N^{3\,2}} = \frac{M_2}{12N^2}(b-a)^3 \qquad M_2 = \max_{x\in\langle a;b\rangle}|f''(x)|$$

Př. $$\int_0^{\frac{\pi}{2}}\sin x\,dx \doteq \frac{\frac{\pi}{8}}{2}\Big[f(x_0)^{0} + 2(0{,}3827+0{,}7071+0{,}9239)+1\Big] \doteq 0{,}9871$$

x_i	$\sin x_i$
0	0
$\pi/8$	0,3827
$\pi/4$	0,7071
$3\pi/8$	0,9239
$\pi/2$	1

Odhad chyby: $$\frac{M_2}{12\cdot N^2}(b-a)^3 = \frac{M_2}{12\cdot 4^2}\left(\frac{\pi}{2}-0\right)^3 = \frac{1}{12\cdot 16}\cdot\frac{\pi^3}{8} = \frac{\pi^3}{1536} \doteq 0{,}0202$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x \quad M_2 = 1$

[$\Rightarrow$ přesná hodnota leží v $(0{,}9871 \pm 0{,}0202)$]

Souhrn NCKV k-tého stupně

k	NÁZEV	JEDNODUCHÉ	SLOŽENÉ	CHYBA
0	OBDÉLNÍKOVÁ METODA	$h \cdot f(x_{i-1})$	$h \sum_{i=1}^{N} f(x_{i-1})$	$M_1 \cdot \frac{(b-a)^2}{2N}$ $M_1 = \max\limits_{x \in \langle a,b \rangle} \lvert f'(x) \rvert$
1	LICHOBĚŽNÍKOVÁ METODA	$\frac{h}{2}(f(x_{i-1}) + f(x_i))$	$\frac{h}{2}\left[f(a) + 2\sum_{i=2}^{N-1} f(x_i) + f(b)\right]$	$\frac{M_2}{12N^2}(b-a)^3$ $M_2 = \max\limits_{x \in \langle a,b \rangle} \lvert f''(x) \rvert$
2	SIMPSONOVA METODA	$\frac{h}{3}(f(x_{i-1}) + 4f(s) + f(x_i))$ s střed $x_{i-1} - x_i$	$\frac{h}{3}\left[f(x_0) + 2\sum_{i=1}^{N-1} f(x_{2i}) + 4\sum_{i=1}^{N} f(x_{2i-1}) + f(x_{2N})\right]$	$\frac{M_4}{\cancel{2880}\,180\, n^4}(b-a)^5$ $M_4 = \max\limits_{x \in \langle a,b \rangle} \lvert f^{IV}(x) \rvert$

Př: $\int_0^{\pi} \sin(x\,dx)$ porovnat OBD a LICH PRAVIDLO $\underline{\varepsilon = 0{,}1}$ $\quad [-\cos x]_0^{\pi} = -\cos\pi + \cos 0 = 1+1 = 2$

kolik musí být n pro přesnost $\varepsilon = 0{,}1$

n=2 $\quad h = \frac{\pi - 0}{2} = \frac{\pi}{2}$

x_i	$f(x_i)$
0	0
$\frac{\pi}{2}$	1
π	0

obd. pravidlo $\int_0^{\frac{\pi}{2}} \sin x\,dx \doteq \frac{\pi}{2}(0+1) = \frac{\pi}{2} \doteq 1{,}5708 = x_2$

lich. pravidlo $\doteq \frac{\pi}{4}(0+2\cdot 1+0) = 2\frac{\pi}{4} \doteq 1{,}5708 = y_2$

Od. chyb: OBD: $M_1 \cdot \frac{(b-a)^2}{2N} = 1 \cdot \frac{\pi^2}{4} = 2{,}4674$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$ $\quad$ LICH: $M_2 \cdot \frac{(b-a)^3}{12N^2} = 1 \cdot \frac{\pi^3}{12\cdot 4} = \frac{\pi^3}{48} \doteq 0{,}646$

n=4 $\quad h = \frac{\pi - 0}{4} = \frac{\pi}{4}$

x_i	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$f(x_i)$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

obd. $\frac{\pi}{4}\left(0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right) \doteq 1{,}8961 \quad x_4$

lich. $\frac{\pi}{8}\left(0 + 2\left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right) + 0\right) = \frac{\pi}{8}\cdot 2(1+\sqrt{2}) \doteq 1{,}8961 \quad y_4$

$|x_4 - x_2| = 0{,}3253$ $\quad$ obd $\quad M_1 \frac{\pi^2}{8} = \frac{\pi^2}{8} = 1{,}2337$

$|y_4 - y_2| = 0{,}3253$ $\quad$ lich $\quad M_2 \frac{\pi^3}{12\cdot 16} = 0{,}1615$

n=8 $\quad h = \frac{\pi - 0}{8} = \frac{\pi}{8}$

x_i	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$	π
$f(x_i)$	0	0,3827 A	0,7071 B	0,9239 C	1 D	0,9239 C	0,7071 B	0,3827 A	0

OBD: $\frac{\pi}{8}(A+B+C+D+C+B+A) \doteq 1{,}9742 \to x_8$

LICH: $\frac{\pi}{16}(0 + 2(2A+2B+2C+D)+0) \doteq 1{,}9742 \to y_8$

$|x_8 - x_4| = 0{,}0781$ $\quad$ OBD: $M_1 \frac{\pi}{16} = \cancel{0{,}1163}$ $0{,}6169$

$|y_8 - y_4| = 0{,}0781$ $\quad$ LICH: $M_2 \frac{\pi^3}{12\cdot 64} = 0{,}0404$

n	h	I(OBD)	I(LICH)	ε(OBD)	ε(LICH)
2	$\pi/2$	1,5708	1,5708	/	/
4	$\pi/4$	0,3257	1,8961	1,2455	0,3253
8	$\pi/8$	1,9742	1,9742	1,6489	0,0781

Př: $\int_{-1}^{3} e^{x^2} dx$ $n = 8$ výpočet: 4DM

a) OBD. PRAVIDLO $h = \frac{3-(-1)}{8} = \frac{1}{2}$

x_i	$f(x_i)$	
-1	2,7183	B
-0,5	1,284	C
0	1	D
0,5	1,284	E
1	2,7183	F
1,5	9,4877	G
2	54,5982	H
2,5	518,0128	I
3	8103,0839	J

$$\int_{-1}^{3} e^{x^2} \doteq_{OP} \frac{1}{2}(B+C+D+E+F+G+H+I) \doteq 295,5517$$

b) LICH. PRAVIDLO $\int_{-1}^{3} e^{x^2} \doteq_{LP} \frac{1}{4}\left[B+2(C+D+E+F+G+H+I)+J\right] \doteq 2320,6431$

c) SIMPSNOVO PRAVIDLO $n=8$

	x_i	$f(x_i)$	
0	-1	2,7183	B
1	-0,75	1,7551	C
2	-0,5	1,284	D
3	-0,25	1,0645	E
4	0	1	
5	0,25	1,0645	E
6	0,5	1,284	D
7	0,75	1,7551	C
8	1	2,7183	B
9	1,25	4,7707	F
10	1,5	9,4877	G
11	1,75	21,3809	H
12	2	54,5982	I
13	2,25	157,925	J
14	2,5	518,0128	K
15	2,75	1924,6511	L
16	3	8103,0839	M

$$h = \frac{3-(-1)}{2 \cdot 8} = \frac{1}{4}$$

$$\int_{-1}^{3} e^{x^2} \doteq_{SP} \frac{1}{12}\left[B+2(D+1+D+B+G+I+K)+4(C+E+E+C+F+H+J+L)+M\right] =$$

$$\doteq 1478,3567$$

Př: $\displaystyle\int_{-\frac{3}{2}}^{\frac{\pi-3}{2}} \sin(2x+3)\,dx$

$N = 8$

$h = \dfrac{\frac{\pi-3}{2}+\frac{3}{2}}{8} = \dfrac{\cancel{\pi-3+3}}{16} = \dfrac{\pi}{16} \rightarrow B$

	x_i		$f(x_i)$	
0	$-\frac{3}{2}$	A	0	
1	$-1{,}3037$	A	0,3827	C
2	$-\frac{3}{2}+2B$ $-1{,}1073$	A	0,7071	D
3	$-\frac{3}{2}+3B$ $-0{,}9110$	A	0,9239	E
4	$-\frac{3}{2}+4B$ $-0{,}7146$	A	1	F
5	$-\frac{3}{2}+5B$ $-0{,}5183$	A	0,9239	E
6	$-\frac{3}{2}+6B$ $-0{,}3219$	A	0,7071	D
7	$-\frac{3}{2}+7B$ $-0{,}1256$	A	0,3827	C
8	$-\frac{3}{2}+8B$ $0{,}0708$	A	0	

OBD: $\doteq \frac{\pi}{16}\left(0+2C+2D+2E+F\right) \doteq 1{,}94$

LICH $\doteq \frac{\pi}{32}\left(0+2(2C+2D+2E+F)+0\right) \doteq 1{,}94$

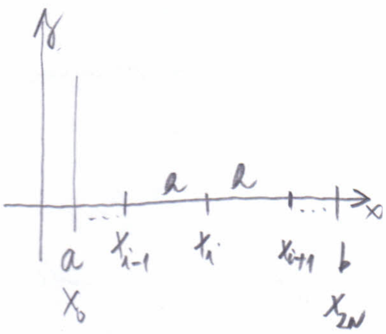
Chyba: OBD $M_1 \dfrac{b-a}{2N} = 1 \cdot \dfrac{\pi}{16} = 0{,}1963$

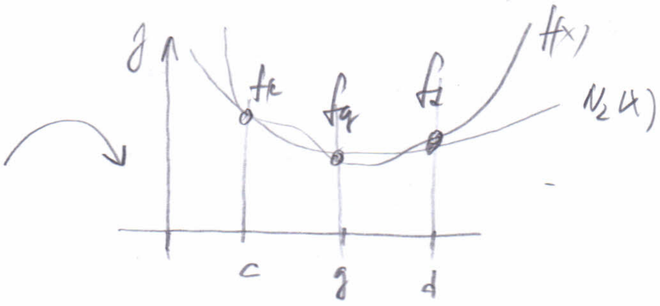
LICH $M_2 \dfrac{(b-a)^3}{12N^2} = 1 \cdot \dfrac{\pi^3}{12 \cdot 64} = 0{,}0404$

SIMS $(n=4) \doteq \frac{\pi}{48}\left(0+2(2D+F)+4(2C+2E)+0\right) = 1{,}7771$

② Simpsonův vzorec

– sudý počet intervalů
– prokládá parabolu (3 body)





Newtonův polynom

x	f(x)	Δ	Δ^2
c	f_c		
		$\frac{f_g-f_c}{g-c}$	
g	f_g		$\frac{\frac{f_d-f_g}{d-g}-\frac{f_g-f_c}{g-c}}{d-c}=\frac{\frac{f_d-f_g}{h}-\frac{f_g-f_c}{h}}{2h}=\frac{\frac{f_d-f_g-f_g+f_c}{h}}{2h}=\frac{1}{2h^2}(f_d-2f_g+f_c)$
		$\frac{f_d-f_g}{d-g}$	
d	f_d		

$$N_2(x)=f_c+(x-c)\cdot\frac{f_g-f_c}{h}+(x-c)(x-g)\cdot\frac{1}{2h^2}(f_d-2f_g+f_c)$$

$$\int_c^d\left[f_c+(x-c)\cdot\frac{f_g-f_c}{h}+(x-c)(x-g)\cdot\frac{1}{h^2}(f_d-2f_d+f_c)\right]dx \overset{\vdots}{\underset{\vdots}{=}} \frac{h}{3}(f_c+4f_g+f_d)$$

viz předn.

$$\int_a^b f(x)\,dx \cong \sum_{i=1}^{N}\int_{x_{2i-2}}^{x_{2i}} L_2(x) = \frac{h}{3}\left(f_0+4f_1+2f_2+4f_3+2f_4+\ldots+4f_{2N-1}+f_{2N}\right]$$

$$\left(f_0+f_{2N}+4\sum f_{\text{sudé}}+2\sum f_{\text{liché}}\right)$$

Př: $\int_0^2(\operatorname{arctg}^2x+x+1)\,dx$ $\quad n=8\quad$ Simpsonova formule

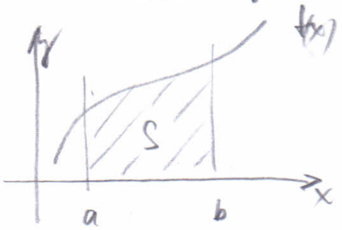
$$h=\frac{b-a}{N}=\frac{2}{16}=\frac{1}{8}=0{,}125$$

i	x_i	f_i	
0	0	1	
1	0,125	1,1405	A
2	0,25	1,3100	B
3	0,375	1,5037	C
4	0,5	1,7150	D
5	0,625	1,9370	E
6	0,75	2,1641	F
7	0,875	2,3917	G
8	1	2,6169	H
9	1,125	2,8376	I
10	1,25	3,0529	J
11	1,375	3,2624	K
12	1,5	3,4659	L
13	1,625	3,6636	M
14	1,750	3,8560	N
15	1,875	4,0432	O
16	2	4,2258	P

$$\int_0^2(\operatorname{arctg}^2x+x+1)\,dx \doteq \frac{h}{3}(f_0+4f_1+2f_2+4f_3+2f_4+4f_5+2f_6+$$
$$+4f_7+2f_8+4f_9+2f_{10}+4f_{11}+2f_{12}+4f_{13}+2f_{14}+4f_{15}+f_{16})=$$
$$\doteq\frac{0{,}125}{3}(1+4A+2B+4C+2D+4E+2F+4G+2H+4I+2J+4K+2L+$$
$$+4M+2N+4O+P)\doteq\underline{5{,}196090702}$$

přesná hodnota:

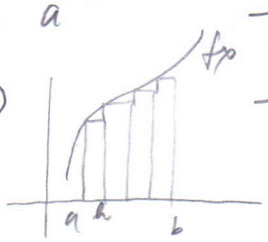
Numerické integrování



$S=\int_a^b f(x)\,dx \rightarrow$ Ⓐ $\int_a^b \varphi(x)\,dx \rightarrow \varphi(x)$ Interpolační polynom

$\rightarrow \varphi(x)$ MNČ

$\rightarrow$ Ⓑ

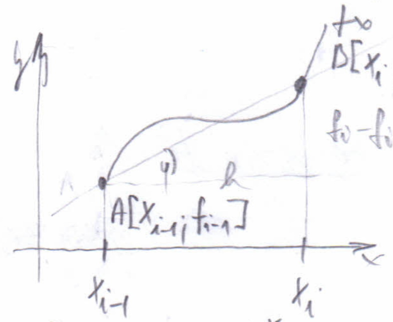


$\rightarrow S=\int_a^b f(x)\,dx \approx \sum_i h\cdot f(x_i)$

obdélníková formule

moc se nepoužívá

1. Newton Cotesova formule (lichoběžníkový vzorec)



$L_1(x):\quad y-y_A=k(x-x_A)$ přímka $k=\operatorname{tg}\varphi$

f_i-f_{i-1} $\quad L_1(x)\Rightarrow y=y_A+k(x-x_A)\Rightarrow f_{i-1}+\dfrac{f_i-f_{i-1}}{h}(x-x_{i-1})$

$$\int_{x_{i-1}}^{x_i} L_1(x)\,dx=\int_{x_{i-1}}^{x_i}\left[f_{i-1}+\frac{f_i-f_{i-1}}{h}(x-x_{i-1})\right]dx=f_{i-1}\int_{x_{i-1}}^{x_i}dx+\frac{f_i-f_{i-1}}{h}\overbrace{\int_{x_{i-1}}^{x_i}(x-x_{i-1})\,dx}^{\frac{(x_i-x_{i-1})^2}{2}}=$$

$$=f_{i-1}\underbrace{(x_i-x_{i-1})}_{h}+\frac{f_i-f_{i-1}}{h}\cdot\frac{h^2}{2}=f_{i-1}\cdot h+\frac{h}{2}(f_i-f_{i-1})=\frac{h}{2}(f_{i-1}+f_i)$$

$$\Rightarrow\int_a^b f(x)\,dx=\sum_{i=1}^{n}\frac{h}{2}(f_{i-1}+f_i)=\frac{h}{2}\left[(f_0+f_1)+(f_1+f_2)+\dots+(f_{n-2}+f_{n-1})+(f_{n-1}+f_n)\right]=$$

$$\underline{=\frac{h}{2}\left(f_0+2f_1+2f_2+\dots+2f_{n-1}+f_n\right)}$$

Př: $\int_{-1}^{3} e^{x^2}\,dx$ $\quad n=8\quad$ Lich. formule

$h=\dfrac{b-a}{n}=\dfrac{3+1}{8}=\dfrac{1}{2}$

i	x_i	f_i	
0	−1	2,7183	A
1	−0,5	1,2840	B
2	0	1	
3	0,5	1,2840	B
4	1	2,7183	A
5	1,5	9,4877	C
6	2	54,5982	D
7	2,5	518,0128	E
8	3	8103,0839	F

$$\int_{-1}^{3} e^{x^2}\,dx=\frac{h}{2}\left(\overset{A}{f_0}+2\overset{B}{f_1}+2f_2+2\overset{B}{f_3}+2\overset{A}{f_4}+2\overset{C}{f_5}+2\overset{D}{f_6}+2\overset{E}{f_7}+\overset{F}{f_8}\right)=$$

$$=\frac{1}{4}\left(3A+4B+2+2C+2D+2E+F\right)\doteq \underline{2320{,}643074}$$

přesná hodnota: 1446,008 ± 0,001

(dělení na 16384 dílků)