Teorie   M III   VAR. 2. 4.1.2022

1. $\frac{2}{5} + \frac{1}{3}i$

2. $y' = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 5x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 8e^{t} \end{pmatrix}$

3. $\sin h\left(\frac{\pi}{2}i\right) = \frac{1}{2}\left(e^{\frac{\pi}{2}i} - e^{-\frac{\pi}{2}i}\right) = \frac{1}{2}\left[\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) - \left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)\right] =$

$= \frac{1}{2}\left[\cancel{\cos\frac{\pi}{2}} + i\sin\frac{\pi}{2} - \cancel{\cos\frac{\pi}{2}} + i\sin\frac{\pi}{2}\right] = \frac{1}{2}\cdot 2i \sin\frac{\pi}{2} = \underline{\underline{i}}$

4. $f(z) = \frac{4z^2+z-2}{z} = 4z+1-2\cdot\left(\frac{1}{z}\right)$

$f'(z) = 4+0-2\cdot\left(-\frac{1}{z^2}\right) = 4 + \frac{2}{z^2}$

$f'(4i) = 4 + \frac{2}{16i^2} = 4 - \frac{1}{8} = \frac{31}{8} = 3\frac{7}{8}$

5. $z(t) = 3 - j + \pi\cdot e^{it} \quad ; t \in \langle 0; 2\pi \rangle$

① $f(z) = \sqrt{2}\cdot e^{i\frac{\pi}{12}}\cdot \sqrt[3]{\frac{-2+2i}{2}} = \sqrt{2}\, e^{i\frac{\pi}{12}}\cdot\sqrt[3]{-1+i}$ ⊜

$|z| = \sqrt{2}$, $\cos\varphi = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$, $\sin\varphi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\Big\}\ \varphi = \frac{3\pi}{4}$

$\sqrt[3]{-1+i} = \sqrt[6]{2}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{3} + i\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right)$

$= \begin{cases} k=0 & \sqrt[6]{2}\, e^{\frac{\pi}{4}} \\ k=1 & \sqrt[6]{2}\, e^{\frac{11\pi}{12}} \\ k=2 & \sqrt[6]{2}\, e^{\frac{19\pi}{12}} \end{cases}$ ⊜ $\sqrt{2}\cdot e^{i\frac{\pi}{12}}$, $\begin{cases} \sqrt[6]{2}\, e^{i\frac{\pi}{4}} = \sqrt[3]{4}\cdot e^{i\frac{\pi}{3}} \\ \sqrt[6]{2}\, e^{i\frac{11\pi}{12}} = \sqrt[3]{4}\cdot e^{i\pi} \\ \sqrt[6]{2}\, e^{i\frac{19\pi}{12}} = \sqrt[3]{4}\cdot e^{i\frac{5}{3}} \end{cases}$

② $\frac{\pi e}{2} j \int_{\Gamma} \sin 3jz\, dz = \frac{\pi e}{2} j\cdot\left(-\frac{1}{3j}\right)\left[\cos 3jz\right]_0^{\pi} = -\frac{\pi e}{6}\left[\cos(3j\pi) - \cos 0\right] =$

$= -\frac{\pi e}{6}\left[\cos 3\pi - \cos 0\right] = -\frac{\pi e}{6_3}\left[-1-1\right] = \underline{\underline{\frac{\pi e}{3}}}$

③ $y'' + y = \sqrt{2}\sin(t\sqrt{2}) \quad ; y(0) = 10; \ y'(0) = 0$

$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \sqrt{2}\,\mathcal{L}\{\sin(t\sqrt{2})\}$

$p^2 Y(p) - 10p - 0 + Y(p) = \sqrt{2}\cdot\frac{\sqrt{2}}{p^2+2}$

$Y(p)\left[p^2+1\right] = \frac{2}{p^2+2} + 10p = \frac{2+10p^3+20p}{p^2+2}$

$Y(p) = \frac{10p^3+20p+2}{(p^2+2)(p^2+1)} = \frac{Ap+B}{p^2+2} + \frac{Cp+D}{p^2+1} = \frac{Ap^3+Ap+Bp^2+B+Cp^3+2Cp+Dp^2+2D}{(p^2+2)(p^2+1)}$

I p^3: $10 = A + C$
II p^2: $0 = B + D$
III p^1: $20 = A + 2C$
IV p^0: $2 = B + 2D$

III−I $C = 10$ $A = 0$
IV−II $D = 2$ $B = -2$

$y(t) = \mathcal{L}^{-1}\left\{\frac{-2}{p^2+2}\right\} + \mathcal{L}^{-1}\left\{\frac{10p+2}{p^2+1}\right\} = -\sqrt{2}\sin(t\sqrt{2}) + 10\mathcal{L}^{-1}\left\{\frac{p}{p^2+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{p^2+1}\right\}$ ⊜

⊜ $\underline{y(t) = -\sqrt{2}\sin(t\sqrt{2}) + 10\cos t + 2\sin t}$

④ $Y(p) = \frac{10p^3+20p+2}{(p^2+2)(p^2+1)}$

$p^2+2 = 0$
$(p+\sqrt{2}i)(p-\sqrt{2}i) = 0$
$p_1 = -\sqrt{2}i$ 1. ř
$p_2 = \sqrt{2}i$ 1. ř
$p^2+1 = 0$
$(p+i)(p-i) = 0$
$p_3 = -i$ 1. ř
$p_4 = i$ 1. ř

$\operatorname{res}_{p=i} Y(p)e^{pt} + \operatorname{res}_{p=-i} Y(p)e^{pt} = 2\operatorname{Re}\operatorname{res}_{p=i} Y(p)e^{pt} = 2\operatorname{Re}\lim_{p\to i} \frac{10p^3+20p+2}{(p^2+2)(p+i)\cancel{(p-i)}}\cancel{(p-i)}e^{pt} = 2\operatorname{Re}\frac{10i^3+20i+2}{(-1+2)(2i)}\cdot e^{it} = 2\operatorname{Re}\frac{10i+2}{2i}e^{it}$ ⊜

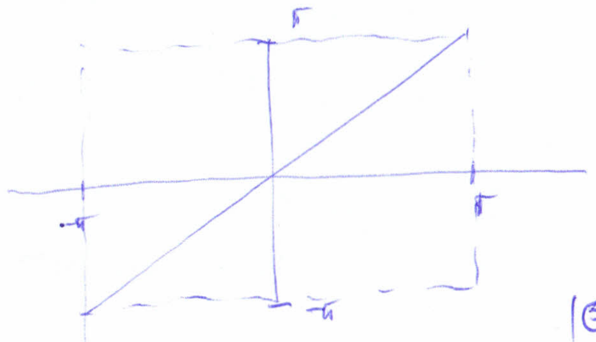
⊜ $2\operatorname{Re}\frac{(10i+2)\cdot i}{2i\cdot i}(\cos t + i\sin t) = 2\operatorname{Re}(5-i)(\cos t + i\sin t) = 2\operatorname{Re}(5\cos t + 5i\sin t - i\cos t + \sin t) = 10\cos t + 2\sin t$

$\operatorname{res}_{p=\sqrt{2}i} Y(p)e^{pt} + \operatorname{res}_{p=-\sqrt{2}i} Y(p)e^{pt} = 2\operatorname{Re}\operatorname{res}_{p=\sqrt{2}i} Y(p)e^{pt} = 2\operatorname{Re}\lim_{p\to\sqrt{2}i}\frac{10p^3+20p+2}{(p+\sqrt{2}i)\cancel{(p-\sqrt{2}i)}(p+i)(p-i)}\cancel{(p-\sqrt{2}i)}e^{pt} = 2\operatorname{Re}\frac{-20\sqrt{2}i+20\sqrt{2}i+2}{2\sqrt{2}i(-1)}e^{\sqrt{2}it}$ ⊟

⊟ $2\operatorname{Re}\frac{2}{2\sqrt{2}i(-1)}\cdot\frac{i}{i}(\cos\sqrt{2}t + i\sin\sqrt{2}t) = 2\operatorname{Re}\frac{i}{\sqrt{2}}(\cos\sqrt{2}t + i\sin\sqrt{2}t) = 2\operatorname{Re}\left(\frac{i}{\sqrt{2}}\cos\sqrt{2}t - \frac{1}{\sqrt{2}}\sin\sqrt{2}t\right) = -\frac{2}{\sqrt{2}}\sin\sqrt{2}t = -\sqrt{2}\sin\sqrt{2}t$

$\underline{y(t) = -\sqrt{2}\sin\sqrt{2}t + 10\cos t + 2\sin t}$

⑤ $f(t)=t$; $t\in\langle-\pi;\pi\rangle$



$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}t\cdot\sin nt\,dt=\frac{1}{\pi}\left[-\frac{t}{n}\cos nt+\frac{1}{n^2}\sin nt\right]_{-\pi}^{\pi}=\frac{1}{\pi}\left[\left(-\frac{\pi}{n}\underbrace{\cos n\pi}_{(-1)^n}+\frac{1}{n^2}\underbrace{\sin n\pi}_{0}\right)-\left(\frac{\pi}{n}\underbrace{\cos(-n\pi)}_{\cos n\pi=(-1)^n}+\frac{1}{n^2}\underbrace{\sin(-n\pi)}_{0}\right)\right]\ominus$$

$$\int t\sin nt\,dt=-\frac{t}{n}\cos nt+\frac{1}{n}\int\cos nt\,dt=-\frac{t}{n}\cos nt+\frac{1}{n^2}\sin nt$$

$u=t$ $u'=1$

$v'=\sin nt$ $v=-\frac{1}{n}\cos nt$

$$\ominus\frac{1}{\pi}\left[-\frac{\pi}{n}(-1)^n-\frac{\pi}{n}(-1)^n\right]=-\frac{\pi}{\pi n}\left[(-1)^n+(-1)^n\right]=-\frac{\cancel{\pi}}{\cancel{\pi}n}\cdot\frac{2(-1)^n}{1}=-\frac{2}{n}(-1)^n=\begin{cases}n\text{ sudé} & -\frac{2}{n}\\ n\text{ liché} & \frac{2}{n}\end{cases}$$

$T=2\pi$; $L=\pi$; lichá fce ; $a_n=0$

$\omega=\frac{2\pi}{2\pi}=1$; spojitá v D

$$t\in\langle-\pi;\pi\rangle\quad t=-2\sum_{n=1}^{\infty}\frac{1}{n}(-1)^n\sin nt=-2\Big[\underbrace{(-\sin t)}_{n=1}+\underbrace{\left(\tfrac{1}{2}\sin 2t\right)}_{n=2}+\underbrace{\left(-\tfrac{1}{3}\sin 3t\right)}_{n=3}+\underbrace{\left(\tfrac{1}{4}\sin 4t\right)}_{n=4}+\underbrace{\left(-\tfrac{1}{5}\sin 5t\right)}_{n=5}+\dots\Big]=$$

$$=2\sin t-\sin 2t+\frac{2}{3}\sin 3t-\frac{1}{2}\sin 4t+\frac{2}{5}\sin 5t+\dots$$