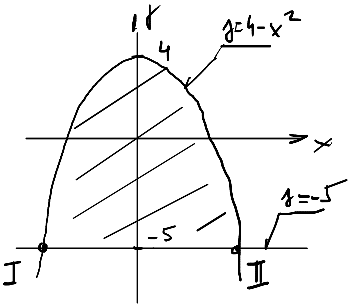
Ukázkový zápočtový test č.3

1. Vypočtěte obsah plochy ohraničené křivkami $y = 4 - x^2$ a $y = -5$.



$I, II:\ 4-x^2=-5$

$x^2-9=0$

$(x-3)(x+3)=0$

$x_1=3$

$x_2=-3$

$$S=\int_{-3}^{3}\left[(4-x^2)-(-5)\right]dx=\int_{-3}^{3}(-x^2+9)\,dx=\left[-\frac{x^3}{3}+9x\right]_{-3}^{3}=$$

$$=\left[\left(-\frac{3^3}{3}+9\cdot 3\right)-\left(-\frac{(-3)^3}{3}+9(-3)\right)\right]=-9+27-9+27=\underline{\underline{36}}$$

2. Řešte počáteční problém pro diferenciální rovnici: $(x+1)y' - 4y = 0;\ y(0) = 1$.

$(x+1)\frac{dy}{dx}=4y$

$\int\frac{dy}{y}=\int\frac{4}{x+1}\,dx$

$\ln|y|=4\ln|x+1|+\ln C$

$\ln|y|=\ln C|x+1|^4$

OŘ: $y=C\cdot(x+1)^4$

PP: $1=C(0+1)^4 \Rightarrow C=1$

PŘ: $\underline{\underline{y=(x+1)^4}}$

3. Je dána funkce $f(x,y) = x^3 + 3x^3y^2 + 2y + xy - 2$. Vypočtěte $\frac{\delta f}{\delta x}(1;1)$ a $\frac{\delta^2 f}{\delta x^2}(1;2)$.

$\frac{\partial f}{\partial x}=3x^2+3\cdot y^2\cdot 3x^2+y=3x^2+9y^2x^2+y$

$\frac{\partial^2 f}{\partial x^2}=6x+9y^2\cdot 2x=6x+18y^2x$

$\frac{\partial f}{\partial x}(1;1)=3\cdot 1^2+9\cdot 1^2\cdot 1^2+1=\underline{\underline{13}}$

$\frac{\partial^2 f}{\partial x^2}(1;2)=6\cdot 1+18\cdot 2^2\cdot 1=\underline{\underline{78}}$

4. Je dána funkce $f(x,y) = x \cdot \sin xy - y \cdot \cos xy$, bod $A\left[1;\frac{\pi}{8}\right]$ a směrový vektor $\vec{s} = (-2;1)$. Vypočtěte derivaci funkce $f(x,y)$ v bodě A ve směru vektoru $\vec{s}$.

jednotkový vektor $\vec{s}_j=\left(-\frac{2}{\sqrt5};\frac{1}{\sqrt5}\right)$

$|\vec{s}|=\sqrt{4+1}=\sqrt5$

$\frac{\partial f}{\partial x}=\sin xy+x\cdot y\cdot\cos xy-\left(y^2(-\sin xy)\right)$

$\frac{\partial f}{\partial y}=x^2\cos xy-\left(\cos xy+y\cdot x(-\sin xy)\right)$

$\operatorname{grad} f(x;y)=\left(\frac{\partial f}{\partial x};\frac{\partial f}{\partial y}\right)=\left(\sin xy+xy\cos xy+y^2\sin xy;\ x^2\cos xy-\cos xy+yx\cdot\sin xy\right)$

$\operatorname{grad} f(A)=\left(\sin\frac{\pi}{8}+\frac{\pi}{8}\cos\frac{\pi}{8}+\left(\frac{\pi}{8}\right)^2\sin\frac{\pi}{8};\ \cancel{\cos\frac{\pi}{8}}-\cancel{\cos\frac{\pi}{8}}+\frac{\pi}{8}\sin\frac{\pi}{8}\right)\doteq(0{,}80;0{,}15)$

$\frac{df}{d\vec{s}}(A)=\operatorname{grad} f(A)\cdot\vec{s}_j=(0{,}80;0{,}15)\cdot\left(-\frac{2}{\sqrt5};\frac{1}{\sqrt5}\right)\doteq 0{,}8\cdot\left(-\frac{2}{\sqrt5}\right)+0{,}15\cdot\frac{1}{\sqrt5}\doteq\underline{\underline{-0{,}65}}$

5. Vypočtěte $\iint_I\ (x + 2y)\,dxdy$, kde $I = \langle 1;3\rangle x\langle 2;5\rangle$.

$$\int_1^3 dx\int_2^5(x+2y)\,dy=\int_1^3 dx\left[xy+\cancel{2}\frac{y^2}{\cancel{2}}\right]_2^5=\int_1^3 dx\left[(5x+25)-(2x+4)\right]=$$

$$=\int_1^3(3x+21)\,dx=\left[3\cdot\frac{x^2}{2}+21x\right]_1^3=\left[\left(3\frac{3^2}{2}+21\cdot 3\right)-\left(3\frac{1^2}{2}+21\cdot 1\right)\right]=$$

$$=3\cdot\frac{9}{2}+63-\frac{3}{2}-21=\underline{\underline{54}}$$