Ukázkový zápočtový test č.1

1. Jsou dány vektory $\vec{a} = (0; 2; 4)$, $\vec{b} = (1; 3; 5)$ a $\vec{c} = (6; 1; 3)$. Vypočtěte $\left(\vec{a} \cdot \vec{b}\right)^2 + (\vec{c} x \vec{a})^2$.

$\vec{a} \cdot \vec{b} = 0 \cdot 1 + 2 \cdot 3 + 4 \cdot 5 = 6 + 20 = 26$ $\quad (\vec{a} \cdot \vec{b})^2 = 26^2 = 676$

$\vec{c} \times \vec{a} = \left\{ \begin{matrix} 6 & 1 & 3 \\ 0 & 2 & 4 \end{matrix} \right\} = \left(\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}; - \begin{vmatrix} 6 & 3 \\ 0 & 4 \end{vmatrix}; \begin{vmatrix} 6 & 1 \\ 0 & 2 \end{vmatrix} \right) = (-2; -24; 12)$

$(\vec{c} \times \vec{a})^2 = (c \times a) \cdot (c \times a) = (-2; -24; 12) \cdot (-2; -24; 12) = 4 + 576 + 144 = 724$

$(\vec{a} \cdot \vec{b})^2 + (\vec{c} \times \vec{a})^2 = 676 + 724 = \underline{\underline{1400}}$

2. Napište vektor $\vec{a} = (6; -1; -3)$, jako lineární kombinaci vektorů $\vec{b} = (2; 1; 0)$, $\vec{c} = (0; 2; 3)$ a $\vec{d} = (2; -1; 0)$.

$\vec{a} = \alpha \vec{b} + \beta \vec{c} + \gamma \vec{d}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 2 & 6 \\ 1 & 2 & -1 & -1 \\ 0 & 3 & 0 & -3 \end{array} \right) :2 \; (-1) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 2 & -2 & -4 \\ 0 & 3 & 0 & -3 \end{array} \right) :2 \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 3 \end{array} \right) :3 \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$\alpha \; \beta \; \gamma$

$\gamma = 1$

$\beta - \gamma = -2$

$\beta - 1 = -2 \Rightarrow \beta = -1$

$\alpha + \gamma = 3$

$\alpha + 1 = 3 \Rightarrow \alpha = 2$

$\vec{a} = 2\vec{b} - \vec{c} + \vec{d}$

3. Vypočtěte: $\lim\limits_{n \to \infty} \ln \frac{3n}{6n+5}$

$\lim\limits_{n \to \infty} \frac{3n}{6n+5} = \lim\limits_{n \to \infty} \frac{\cancel{n} \cdot 3}{\cancel{n}(6 + 5 \cdot \frac{1}{n})} = \frac{3}{6} = \frac{1}{2}$, $\quad \frac{1}{n} \to 0$

$\lim\limits_{n \to \infty} \ln \frac{3n}{6n+5} = \lim\limits_{n \to \infty} \ln \frac{1}{2} = \ln 2^{-1} = -\ln 2 \doteq -0{,}69$

4. Vypočítejte: $A = 3 \cdot \sqrt{3} \cdot \sqrt[4]{3} \cdot \sqrt[8]{3} \cdot \ldots$

$A = 3^1 \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{1}{4}} \cdot 3^{\frac{1}{8}} \cdot \ldots = 3^{\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)} = 3^2 = 9$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

5. Je dána funkce $f: y = 2x + 3\sqrt{x} - \sqrt[3]{x^2} + \frac{2}{\sqrt{x}}$. Vypočtěte $y'(1)$.

$y = 2x + 3 \cdot x^{\frac{1}{2}} - x^{\frac{2}{3}} + 2 \cdot x^{-\frac{1}{2}}$

$y' = 2 + 3 \cdot \frac{1}{2} x^{-\frac{1}{2}} - \frac{2}{3} x^{-\frac{1}{3}} + 2 \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$

$y' = 2 + \frac{3}{2 \cdot \sqrt{x}} - \frac{2}{3\sqrt[3]{x}} - \frac{1}{\sqrt{x^3}}$

$y'(1) = 2 + \frac{3}{2 \cdot 1} - \frac{2}{3 \cdot 1} - \frac{1}{1} = 2 + \frac{3}{2} - \frac{2}{3} - 1 = \frac{11}{6} = 1\frac{5}{6} \doteq 1{,}83$