

1. Vypočtěte $\int_0^1 \int_{2x}^2 (x-y) dy dx$

$$\int_0^1 \int_{2x}^2 (x-y) dy dx = \int_0^1 \left[xy - \frac{y^2}{2} \right]_{2x}^2 dx = \int_0^1 [(2x-2) - (2x^2 - 2x^2)] dx \equiv$$

$$\equiv \int_0^1 (2x-2) dx = \left[x^2 - 2x \right]_0^1 = (1-2) - 0 = \underline{\underline{-1}}$$

2. Vypočtěte limitu, pokud k výpočtu použijete některé pravidlo, zdůvodněte proč.

$$\lim_{x \rightarrow 7} \frac{\sqrt{-2x+18} - 2}{x+1-8e^{-x+7}}$$

$$\lim_{x \rightarrow 7} \frac{\sqrt{-2x+18} - 2}{x+1-8 \cdot e^{-x+7}} = \frac{0}{0} \text{'' L'H} = \lim_{x \rightarrow 7} \frac{\frac{1}{2\sqrt{-2x+18}} \cdot (-2)}{1-8 \cdot e^{-x+7} \cdot (-1)} \equiv$$

$$\equiv \frac{-\frac{1}{\sqrt{4}}}{1-8 \cdot 1 \cdot (-1)} = \frac{-\frac{1}{2}}{9} = -\frac{1}{18}$$

3. Napište rovnice tečny a normály funkce $f(x) = (-x^2 + 10x - 9) \cdot 3^x + 4$ v dotykovém bodě $T = [1; ?]$

$$y_T = (-1^2 + 10 \cdot 1 - 9) \cdot 3^1 + 4 = (-1 + 10 - 9) \cdot 3 + 4 = 4 \quad T[1; 4]$$

$$(x=1)$$

$$y' = (-2x + 10) \cdot 3^x + (-x^2 + 10x - 9) \cdot 3^x \cdot \ln 3$$

$$k_t = y'(T) = (-2 \cdot 1 + 10) \cdot 3 + (-1 + 10 - 9) \cdot 3 \cdot \ln 3 = 8 \cdot 3 = 24$$

$$k_n = 24 \Rightarrow k_n = -\frac{1}{24}$$

$$t: y - 4 = 24(x - 1)$$

$$y - 4 = 24x - 24$$

$$t: \underline{24x - y - 20 = 0}$$

$$n: y - 4 = -\frac{1}{24}(x - 1)$$

$$24y - 96 = -x + 1$$

$$n: \underline{x + 24y - 97 = 0}$$

4. Řešte počáteční problém pro diferenciální rovnici

$$y' - 3x^2y = 0, y(0) = 2.$$

$$\frac{dy}{dx} = 3x^2y$$

$$\int \frac{dy}{y} = 3 \int x^2 dx$$

$$\ln(y) = 1 \cdot \frac{x^3}{3} + C$$

$$OE: \underline{y = e^{x^3 + C} = C e^{x^3}}$$

$$PD: x=0 \Rightarrow y=2$$

$$2 = C \cdot e^0 \Rightarrow C=2$$

$$PE: \underline{y = 2 \cdot e^{x^3}}$$

5. Je dána funkce $f(x; y) = \ln(x^2 + 3y) + y^2 \cdot \sin x + e^{-5}$ a bod $M = \left[0; \frac{1}{2}\right]$. Vypočítejte

$\frac{\partial f}{\partial x}(M)$ a $\frac{\partial f}{\partial y}(M)$.

$$\frac{\partial f}{\partial x} = \frac{1}{x^2 + 3y} \cdot 2x + y^2 \cos x + 0$$

$$\frac{\partial f}{\partial x}(M) = \frac{1}{0^2 + \frac{3}{2}} \cdot 2 \cdot 0 + \left(\frac{1}{2}\right)^2 \cdot \underbrace{\cos 0}_{\frac{1}{1}} = \frac{1}{4}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x^2 + 3y} \cdot 3 + \sin x \cdot 2y + 0$$

$$\frac{\partial f}{\partial y}(M) = \frac{1}{0^2 + \frac{3}{2}} \cdot 3 + \sin 0 \cdot 2 \cdot \frac{1}{2} =$$

$$= \frac{3}{\frac{3}{2}} = \frac{1}{1} \cdot \frac{2}{1} = 2$$

6. Určete maximální intervaly konvexity a konkávnosti funkce $y = \frac{-4x^2 - 3}{x^2 + 3}$.

$D_f: x \in \mathbb{R}$

$$y' = \frac{-8x(x^2 + 3) - (-4x^2 - 3) \cdot 2x}{(x^2 + 3)^2} = \frac{-8x^3 - 24x + 8x^3 + 6x}{(x^2 + 3)^2} = -\frac{18x}{(x^2 + 3)^2}$$

$$y'' = \frac{-18(x^2 + 3) - (-18x) \cdot 2(x^2 + 3) \cdot 2x}{(x^2 + 3)^4} = \frac{-18x^2 - 54 + 72x^2}{(x^2 + 3)^3} = \frac{54x^2 - 54}{(x^2 + 3)^3}$$

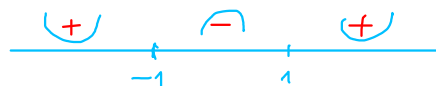
$$y'' = 0 \Leftrightarrow 54x^2 - 54 = 0 \quad | :54$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$\downarrow \quad \downarrow$$

$$I_{01} = -1 \quad I_{02} = 1$$



konvexní na $x \in (-\infty; -1)$ a na $x \in (1; \infty)$

konkávní na $x \in (-1; 1)$