1. Vypočtěte $\int_0^1 \int_{2x}^2 (x-y)\,dy\,dx$

$$\int_0^1 \int_{2x}^2 (x-y)\,dy\,dx = \int_0^1 \left[xy - \frac{y^2}{2}\right]_{2x}^2 dx = \int_0^1 \left[(2x-2) - (\cancel{2x^2} - \cancel{2x^2})\right] dx \ominus$$

$$\ominus \int_0^1 (2x-2)\,dx = \left[\cancel{2}\frac{x^2}{\cancel{2}} - 2x\right]_0^1 = (1-2) - 0 = -1$$

2. Vypočtěte limitu, pokud k výpočtu použijete některé pravidlo, zdůvodněte proč.

$$\lim_{x\to 7} \frac{\sqrt{-2x+18} - 2}{x+1-8e^{-x+7}}$$

$$\lim_{x\to 7} \frac{\sqrt{-2x+18} - 2}{x+1-8\cdot e^{-x+7}} = \text{"}\frac{0}{0}\text{"} \; LP = \lim_{x\to 7} \frac{\frac{1}{1\cancel{2}\sqrt{-2x+18}} \cdot \overset{(-1)}{(-\cancel{2})}}{1 - 8\cdot e^{-x+7}\cdot(-1)} \ominus$$

$$\ominus \frac{-\frac{1}{\sqrt{4}}}{1-8\cdot 1\cdot(-1)} = \frac{-\frac{1}{2}}{9} = -\frac{1}{18}$$

3. Napište rovnice tečny a normály funkce $f(x) = (-x^2 + 10\,x - 9) \cdot 3^x + 4$ v dotykovém bodě $T = [1; ?]$

$y_T = (-1^2 + 10 \cdot 1 - 9) \cdot 3^1 + 4 = (-1 + 10 - 9) \cdot 3 + 4 = 4$ $\quad T[1;4]$
$(x=1)$

$y' = (-2x + 10) \cdot 3^x + (-x^2 + 10x - 9) \cdot 3^x \cdot \ln 3$

$k_t = y'_{(T)} = (-2 \cdot 1 + 10) \cdot 3 + (-\underbrace{1 + 10 - 9}_{0}) \cdot 3 \cdot \ln 3 = 8 \cdot 3 = 24$
$x = 1$

$k_t = 24$
$\Rightarrow k_n = -\frac{1}{24}$

$t: \; y - 4 = 24(x - 1)$
$y - 4 = 24x - 24$
$t: \; \underline{\underline{24x - y - 20 = 0}}$

$n: \; y - 4 = -\frac{1}{24}(x - 1)$
$24y - 96 = -x + 1$
$n: \; \underline{\underline{x + 24y - 97 = 0}}$

4. Řešte počáteční problém pro diferenciální rovnici

$$y' - 3x^2 y = 0, y(0) = 2.$$

$\frac{dy}{dx} = 3x^2 y$

$\int \frac{dy}{y} = 3 \int x^2 dx$

$\ln|y| = \cancel{3}^{1} \cdot \frac{x^3}{\cancel{3}_{1}} + c$

$OŘ: \; \underline{\underline{y = e^{x^3 + c} = C e^{x^3}}}$

$PP: \; x = 0 \Rightarrow y = 2$
$2 = C \cdot e^0 \Rightarrow C = 2$

$PŘ: \; \underline{\underline{y = 2 \cdot e^{x^3}}}$

5. Je dána funkce $f(x;y) = \ln(x^2 + 3y) + y^2 \cdot \sin x + e^{-5}$ a bod $M = \left[0;\frac{1}{2}\right]$. Vypočtěte $\frac{\partial f}{\partial x}(M)$ a $\frac{\partial f}{\partial y}(M)$.

$$\frac{\partial f}{\partial x} = \frac{1}{x^2+3y} \cdot 2x + y^2 \cos x + 0$$

$$\underset{\substack{x=0\\ y=\frac{1}{2}}}{\frac{\partial f}{\partial x}(M)} = \underbrace{\frac{1}{0^2+\frac{3}{2}} \cdot 2 \cdot 0}_{0} + \left(\frac{1}{2}\right)^2 \cdot \underbrace{\cos 0}_{1} = \frac{1}{4}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x^2+3y} \cdot 3 + \sin x \cdot 2y + 0$$

$$\underset{\substack{x=0\\ y=\frac{1}{2}}}{\frac{\partial f}{\partial y}(M)} = \frac{1}{0^2+\frac{3}{2}} \cdot 3 + \underbrace{\sin 0 \cdot 2 \cdot \frac{1}{2}}_{0} =$$

$$= \frac{3}{\frac{3}{2}} = \cancel{3}^{1} \cdot \frac{2}{\cancel{3}_{1}} = 2$$

6. Určete maximální intervaly konvexity a konkávity funkce $y = \frac{-4x^2-3}{x^2+3}$.

$D_f: x \in \mathbb{R}$

$$y' = \frac{-8x(x^2+3) - (-4x^2-3)\cdot 2x}{(x^2+3)^2} = \frac{\cancel{-8x^3} - 24x + \cancel{8x^3} + 6x}{(x^2+3)^2} = -\frac{18x}{(x^2+3)^2}$$

$$y'' = \frac{-18(x^2+3)^{\cancel{2}\,1} - (-18x)\cdot 2\cancel{(x^2+3)}\cdot 2x}{(x^2+3)^{\cancel{4}\,3}} = \frac{-18x^2 - 54 + 72x^2}{(x^2+3)^3} = \frac{54x^2-54}{(x^2+3)^3}$$

$y'' = 0 \Leftrightarrow 54x^2 - 54 = 0 \quad /:54$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$IB_1 = -1 \quad IB_2 = 1$

$+$	-1	$-$	1	$+$

konvexní na $x \in (-\infty; -1\rangle$ a na $x \in \langle 1; \infty)$

konkávní na $x \in \langle -1; 1\rangle$