Vypočtěte limitu posloupnosti:

$$\lim_{n\to\infty}\left[\frac{(2n-1)(3-5n)}{3n^2}-\frac{3^n+5\cdot 5^n}{3\cdot 5^n-2\cdot 3^n}\right]=\lim_{n\to\infty}\frac{6n-10n^2-3+5n}{3n^2}-\lim_{n\to\infty}\frac{5^n\left(\frac{3^n}{5^n}+5\right)}{5^n\left(3-2\frac{3^n}{5^n}\right)}=$$

$$=\lim_{n\to\infty}\frac{-10n^2+11n-3}{3n^2}-\lim_{n\to\infty}\frac{\cancel{5^n}\left(\left(\frac{3}{5}\right)^n+5\right)}{\cancel{5^n}\left(3-2\left(\frac{3}{5}\right)^n\right)}=-\frac{10}{3}-\frac{5}{3}=-\frac{15}{3}=-5$$

Je dána funkce $f(x)=\frac{3x^2-x-6}{e^x}$. Vypočtěte $f'(2)$ a výsledek zaokrouhlete na 2 desetinná místa.

$D_f: x\in\mathbb{R}$

$$f'(x)=\frac{(6x-1)e^x-(3x^2-x-6)\cdot e^x}{(e^x)^2}=\frac{6xe^x-e^x-3x^2e^x+xe^x+6e^x}{(e^x)^2}=$$

$$=\frac{-3x^2e^x+7xe^x+5e^x}{(e^x)^2}=\frac{\cancel{e^x}(-3x^2+7x+5)}{(e^x)^{\cancel{2}}}=\frac{-3x^2+7x+5}{e^x}$$

$$f'(2)=\frac{-3\cdot 2^2+7\cdot 2+5}{e^2}=\frac{7}{e^2}\doteq 0{,}95$$

Rozhodněte, zda existují asymptoty funkce $f(x)=2x-\frac{3}{x+4}$ a všechny nalezněte.

$D_f: x\in\mathbb{R}-\{-4\}$

① $\lim_{x\to -4^+}\left(2x-\frac{3}{x+4}\right)=\text{"}-8-\frac{3}{0^+}\text{"}=\text{"}-8-\infty\text{"}=-\infty$

svislá asymptota

$x+4=0$

② $\lim_{x\to\pm\infty}\left(2x-\frac{3}{x+4}\right)=\text{"}\infty-0\text{"}=\infty$ horizontální asymptota $\nexists$

$-\infty+0=-\infty$

③ $y=kx+q$ $\quad k=\lim_{x\to\pm\infty}\left(\frac{2x-\frac{3}{x+4}}{x}\right)=\lim_{x\to\pm\infty}\left(2-\underbrace{\frac{3}{x^2+4x}}_{0}\right)=2$

$q=\lim_{x\to\pm\infty}\left(\cancel{2x}-\frac{3}{x+4}-\cancel{2x}\right)=0$ $\quad$ ASS: $y=2x$

Napište rovnici tečny a normály ke grafu funkce $f(x) = (3-x)\cdot e^{2x}$
v dotykovém bodě $T = [0; ?]$.

$y_T = (3-0)\cdot e^{2\cdot 0} = 3\cdot e^0 = 3 \quad T[0;3]$

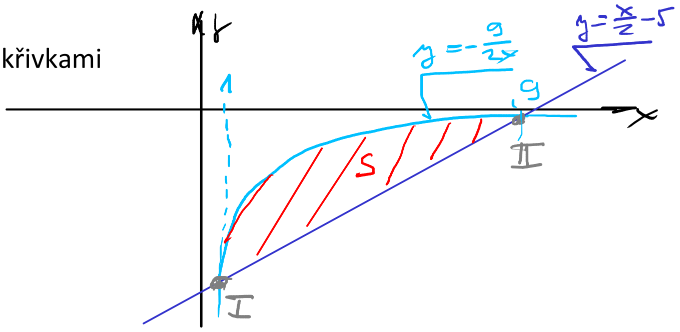
$y' = (-1)\cdot e^{2x} + (3-x)e^{2x}\cdot 2 \quad k_t = y'_{(0)} = (-1)\cdot e^0 + 3\cdot e^0\cdot 2 = -1+6 = 5$

$k_n = -\frac{1}{k_t} = -\frac{1}{5}$

tečna: $y - y_T = k_t(x - x_T)$
$y - 3 = 5(x-0)$
$t: 5x - y + 3 = 0$

normála: $y - y_T = k_n(x - x_T)$
$y - 3 = -\frac{1}{5}x \;/\cdot 5$
$5y - 15 = -x$
$n: x + 5y - 15 = 0$

Vypočtěte obsah plochy rovinného obrazce ohraničeného křivkami
$2xy = -9$ a $x - 2y = 10$




$2xy = -9$
$y = -\frac{9}{2x}$

$x - 2y = 10$
$-2y = 10 - x$
$y = \frac{x}{2} - 5$

$I, II: \; -\frac{9}{2x} = \frac{x}{2} - 5 \;/\cdot 2x$
$-9 = x^2 - 10x$
$x^2 - 10x + 9 = 0$
$(x-1)(x-9) = 0$
$x_1 = 1 \quad x_2 = 9$

$S = \int_1^9 \left[\left(-\frac{9}{2x}\right) - \left(\frac{x}{2} - 5\right)\right] dx = \int_1^9 \left(-\frac{9}{2}\cdot\frac{1}{x} - \frac{1}{2}x + 5\right) dx =$

$= \left[-\frac{9}{2}\ln|x| - \frac{1}{2}\frac{x^2}{2} + 5x\right]_1^9 = \left(-\frac{9}{2}\ln 9 - \frac{1}{4}\cdot 81 + 5\cdot 9\right) - \left(-\frac{9}{2}\underbrace{\ln 1}_{0} - \frac{1}{4} + 5\right) =$

$= -\frac{9}{2}\ln 9 - \frac{81}{4} + 45 - 0 + \frac{1}{4} - 5 = -\frac{9}{2}\ln 9 - \underbrace{\frac{80}{4}}_{20} + 40 = -\frac{9}{2}\ln 9 + 20 \doteq 10{,}11$

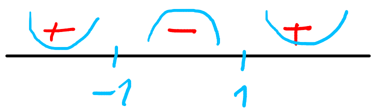
Určete maximální intervaly konvexity a konkávity funkce $y = \frac{-4x^2-3}{x^2+3}$.

$D_f: x \in R$

$y' = \frac{(-8x)(x^2+3) - (-4x^2-3)\cdot 2x}{(x^2+3)^2} = \frac{-\cancel{8x^3} - 24x + \cancel{8x^3} + 6x}{(x^2+3)^2} = \frac{-18x}{(x^2+3)^2} \quad D_{f'}: x \in R$

$y'' = \frac{(-18)(x^2+3)^{\cancel{2}} - (-18x)\cancel{2(x^2+3)}\cdot 2x}{(x^2+3)^{\cancel{4}\,3}} = \frac{-18x^2 - 54 + 72x^2}{(x^2+3)^3} = \frac{54x^2 - 54}{(x^2+3)^3} \quad D_{f''}: x \in R$

$y'' = 0: \; 54x^2 - 54 = 0$
$x^2 - 1 = 0$
$(x-1)(x+1) = 0$
$x_1 = 1 \quad x_2 = -1$
$IB_1 \quad IB_2$




konvexní: $x \in (-\infty; -1)$ a na $\langle 1; \infty)$
konkávní: $x \in \langle -1; 1 \rangle$