XAMAT, YAMAT

Cvičení 8

Diferenciální rovnice 1. řádu

1. Metodou separace proměnných řešte diferenciální rovnice:

a) $(x+1)y' - 4y = 0$ $\quad [y = C(x+1)^4]$

b) $3y' - sin^3x \cdot cosx = 0$ $\quad [y = \frac{sin^4x}{12} + C]$

c) $xy' - 5ln^4x = 0$ $\quad [y = ln^5x + C]$

d) $2\sqrt{x} \cdot y' - cos^2y = 0$ $\quad [y = arctg(\sqrt{x} + C)]$

e) $e^y(x^2+1)y' - x^2 = 0$ $\quad [y = ln|x - arctgx + C|]$

f) $(xlnx + 2x)y' = y$ $\quad [y = C(lnx + 2)]$

g) $y' = xsinx \cdot 2y$ $\quad [y = Ce^{2(sinx - xcosx)}]$

2. Metodou separace proměnných řešte diferenciální rovnice:

a) $y' + y \cdot tgx = 0$ $\quad [y = Ccosx]$

b) $y' + 2y = 0$ $\quad [y = Ce^{-2x}]$

c) $xy' + 2y = 0$ $\quad [y = \frac{C}{x^2}]$

d) $y' - 3x^2y = 0$ $\quad [y = Ce^{x^3}]$

e) $y'sinx - y\,cosx = 0$ $\quad [y = Csinx]$

f) $y' - y\,cosx = 0$ $\quad [y = Ce^{sinx}]$

g) $y' - \frac{y}{\sqrt{1-x^2}} = 0$ $\quad [y = Ce^{arcsinx}]$

h) $y'y^7 - \frac{1}{x} = 0$ $\quad [y = \sqrt[8]{8(ln|x| + C)}]$

i) $1 + y^2 - y'x^{10} = 0$ $\quad [y = tg(\frac{x^{-9}}{-9} + C)]$

j) $cos^2y - y'x^5 = 0$ $\quad [y = arctg(\frac{x^{-4}}{-4} + C)]$

3. Metodou separace proměnných řešte diferenciální rovnice:

a) $y' = cos\,x$ $\quad [y = sinx + C]$

b) $y' = \sqrt{1-y^2}$ $\quad [y = \sin(x + C)\,; y = \pm 1]$

c) $x^2y' + y^2 = 0$ $\quad [y = -\frac{x}{1+Cx}; y = 0]$

d) $yy' + x = 0$ [$y^2 = C^2 - x^2$]

e) $xy' = \sqrt{x}cos^2y$ [$tg\ y = 2\sqrt{x} + C; y = \frac{\pi}{2} + k\pi$]

f) $2y'\sqrt{x} = y$ [$y = Ce^{\sqrt{x}}$]

g) $y + y'cotgx = 0$ [$y = Ccosx$]

h) $xy' = y - 1$ [$y = Cx + 1$]

i) $y - xy' = 3xy$ [$y = Cxe^{-3x}$]

j) $2xyy' = y^2 - 3$ [$y^2 = Cx + 3$]

k) $y'e^{6-5x} = y^2$ [$y = \frac{-5}{e^{5x-6}+C}; y = 0$]

l) $\frac{y'}{cosx} - (5x + 2) \cdot y^2 = 0$ [$y = \frac{-1}{(5x+2)sinx+5cosx+C}; y = 0$]

4. Metodou variace konstanty najděte obecné řešení diferenciální rovnice:

a) $y' + 3y = \frac{x^2+5x+1}{e^{3x}}$ [$y = (\frac{1}{3}x^3 + \frac{5}{2}x^2 + x + C)e^{-3x}$]

b) $y' - y \cdot tg\ x = \frac{x+1}{cosx}$ [$y = (\frac{1}{2}x^2 + x + C)\frac{1}{cosx}$]

c) $y' - 3x^2y = 3x^2e^{x^3}$ [$y = (x^3 + C)e^{x^3}$]

d) $-3y' + 30y = 8e^{9x}$ [$y = Ce^{10x} + \frac{8}{3}e^{9x}$]

e) $y' - ycosx = e^{sinx}$ [$y = (x + C)e^{sinx}$]

f) $xy' - 2y = x^2lnx$ [$y = \frac{1}{2}x^2ln^2x + Cx^2$]

g) $y' - y \cdot cotg\ x = (x + 1)sinx$ [$y == (\frac{1}{2}x^2 + x + C)sinx$]

h) $2y' - 16y = 9e^{10x}$ [$y = Ce^{8x} + \frac{9}{4}e^{10x}$]

i) $y' + xy = x$ [$y = 1 + Ce^{-\frac{x^2}{2}}$]

j) $y' + 2y = e^{-x}$ [$y = Ce^{-2x} + e^{-x}$]