

XAMAT, YAMAT

Cvičení 7

Určitý integrál a jeho použití

1. $\int_1^2 2^{3x-3} dx$ $\left\{ \frac{7}{3 \ln 2} \right\}$
2. $\int_0^3 \sqrt{9-3x} dx$ $\{6\}$
3. $\int_0^{4\pi} \frac{dx}{\sin^2(\frac{\pi+x}{6})}$ $\{12\sqrt{3}\}$
4. $\int_0^3 2 dx$ $\{6\}$
5. $\int_2^4 (x+1) dx$ $\{8\}$
6. $\int_{-1}^1 (2t-1) dt$ $\{-2\}$
7. $\int_{-2}^2 |x| dx$ $\{4\}$
8. $\int_0^4 5x^2 dx$ $\left\{ \frac{320}{3} \right\}$
9. $\int_0^1 3^x dx$ $\left\{ \frac{2}{\ln 3} \right\}$
10. $\int_2^5 x^3 dx$ $\left\{ \frac{609}{4} \right\}$
11. $\int_0^2 (1+2x)^6 dx$ $\left\{ 5580 \frac{2}{7} \right\}$
12. $\int_{12}^4 \frac{2}{y} dy$ $\{-2 \ln 3\}$
13. $\int_0^{2\pi} (2 \sin x - \cos x) dx$ $\{0\}$
14. $\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x} dx$ $\{1\}$
15. $\int_0^1 (4^x + 3x) dx$ $\left\{ 3 \left(\frac{1}{2} + \frac{1}{\ln 4} \right) \right\}$
16. $\int_0^{\frac{\pi}{2}} \cos^2 t \cdot \sin t dt$ $\left\{ \frac{1}{3} \right\}$
17. $\int_0^{\frac{\pi}{2}} \sin(2x) dx$ $\{1\}$
18. $\int_0^1 x \cdot e^x dx$ $\{1\}$
19. $\int_{-1}^0 2^x (2x+2) dx$ $\left\{ \frac{-1+\ln 4}{\ln^2 2} \right\}$
20. $\int_1^e \frac{1+\ln x}{x} dx$ $\left\{ \frac{3}{2} \right\}$
21. $\int_1^2 \frac{1}{x^2} \cdot e^{\frac{1}{x}} dx$ $\{e - \sqrt{e}\}$
22. $\int_0^{\frac{\pi}{2}} x \cdot \cos x dx$ $\left\{ \frac{\pi}{2} - 1 \right\}$
23. $\int_0^{\frac{\pi}{2}} e^{2x} \cdot \sin x dx$ $\left\{ \frac{1}{5} (2e^\pi + 1) \right\}$

$$24. \int_0^1 \operatorname{arctg} u \, du \quad \left\{ \frac{\pi}{4} - \ln \sqrt{2} \right\}$$

$$25. \int_0^1 \frac{\operatorname{arctg} x}{1+x^2} \, dx \quad \left\{ \frac{\pi^2}{32} \right\}$$

$$26. \int_1^e \ln^2 x \, dx \quad \{e - 2\}$$

Vypočítejte obsah rovinného obrazce ohraničeného křivkami:

$$a) \ y = 2 - \frac{1}{3}x^2; x + y = 2 \quad \left\{ \frac{3}{2} \right\}$$

$$b) \ y = 4 - x^2; y = -5 \quad \{36\}$$

$$c) \ y = 4 - x^2; y = 3x \quad \left\{ \frac{125}{6} \right\}$$

$$d) \ y = \frac{1}{2}x^2 - 1; x + y = 3 \quad \{18\}$$

$$e) \ y = \frac{6}{x}; y = 7 - x \quad \left\{ \frac{35}{2} - 6 \ln 6 \right\}$$

$$f) \ y = x; y = \frac{1}{x}; x = 2 \quad \left\{ \frac{3}{2} - \ln 2 \right\}$$

$$g) \ y = x^3; y = 4x \quad \{8\}$$

$$h) \ y = x^2 + 1; y = 2x^2 \quad \left\{ \frac{4}{3} \right\}$$

$$i) \ y = 3 - x; y = 2\sqrt{x}; y = 0 \quad \left\{ \frac{10}{3} \right\}$$

$$j) \ y = \ln x; y = 2; x = e^4 \quad \{e^4 + e^2\}$$

$$k) \ y = \frac{x^2}{3}; y = 1 + \frac{2}{3}x; y = 0; x = 6 \quad \{15\}$$

$$l) \ y = 4x - x^2; y = 3x - 6; y = 0; \text{pro } x \geq 2 \quad \left\{ \frac{19}{6} \right\}$$

$$m) \ y = x; y = \frac{4}{x}; x = 1; x = 6; y = 0 \quad \left\{ \frac{3}{2} + 4 \ln 3 \right\}$$

$$n) \ y = x + 4; y = x^2 + 2; y = 0; x = 2 \quad \left\{ \frac{27}{2} \right\}$$

$$o) \ y = e^x; y = e^{2-x}; x = 0; x = 3; y = 0 \quad \{2e - 1 - e^{-1}\}$$