XAMAT, YAMAT

Cvičení 2

Posloupnosti

1. Vypočtěte limity posloupostí:
   a) $\lim_{n\to\infty} \frac{n^2+2n+5}{n^3+1}$ $\{0\}$
   b) $\lim_{n\to\infty} \frac{n^3+1}{n^2+2n+3}$ $\{\infty\}$
   c) $\lim_{n\to\infty} \frac{n^3}{2n^3-n^2+2}$ $\left\{\frac{1}{2}\right\}$
   d) $\lim_{n\to\infty} \frac{1-n^2}{3n^2+n-1}$ $\left\{-\frac{1}{3}\right\}$
   e) $\lim_{n\to\infty} \frac{7n+5}{4-2n}$ $\left\{-\frac{7}{2}\right\}$
   f) $\lim_{n\to\infty} \frac{5n^2-4n+7}{17n^2+n+6}$ $\left\{\frac{5}{17}\right\}$
   g) $\lim_{n\to\infty} \frac{(n+1)(n+2)(n+3)}{n^4+1}$ $\{0\}$
2. Vypočtěte limity posloupostí:
   a) $\lim_{n\to\infty} \left(3+\frac{4}{3n}\right)$ $\{3\}$
   b) $\lim_{n\to\infty} \left(\frac{1+2+\cdots+n}{n+2}-\frac{n}{2}\right)$ $\left\{-\frac{1}{2}\right\}$
   c) $\lim_{n\to\infty} \left(\frac{n^5+1}{2n^5+3n}\right)^4$ $\left\{\frac{1}{16}\right\}$
   d) $\lim_{n\to\infty} 4^{\frac{6n+2}{3n-4}}$ $\{16\}$
   e) $\lim_{n\to\infty} \ln\frac{3n}{6n+5}$ $\{-ln2\}$
   f) $\lim_{n\to\infty} \left[\left(1+\frac{3}{n}\right)\left(2-\frac{4}{n}\right)^2\left(\frac{5}{n^2}-1\right)\right]$ $\{-4\}$
   g) $\lim_{n\to\infty} \left(3-\frac{5}{4n}\right)^{100}$ $\{3^{100}\}$
   h) $\lim_{n\to\infty} \left[\left(\frac{2n^2+1}{3-n^2}\right)\left(1+\frac{2}{n}+\frac{3n-1}{5n^2+4}\right)\right]$ $\{-2\}$
   i) $\lim_{n\to\infty} \left(\sqrt[n]{4}-16\right)$ $\{-15\}$
   j) $\lim_{n\to\infty} \frac{n!-(n+2)!}{(n+1)!}$ $\{-\infty\}$

3. Vypočtěte limity posloupostí:
   a) $\lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n+1}$ $\{1\}$
   b) $\lim_{n\to\infty} \frac{\sqrt[3]{n^3+2n-1}}{n+2}$ $\{1\}$
   c) $\lim_{n\to\infty} \frac{4n^2+\sqrt{n^3+1}}{n^2+n+1}$ $\{4\}$
   d) $\lim_{n\to\infty} \frac{2^n-1}{2^n+1}$ $\{1\}$
   e) $\lim_{n\to\infty} \frac{2^n+3^n}{2^n-3^n}$ $\{-1\}$

4. Vypočtěte limity posloupostí:
   a) $\lim_{n\to\infty} \frac{n^2+2n+3}{n^3+1}$ $\{0\}$
   b) $\lim_{n\to\infty} \frac{n^3-1}{n^2+n}$ $\{\infty\}$

c) $\lim_{n\to\infty} \frac{(2n+1)(3n+3)(3n-8)}{1-n^2+n^3}$ $\{18\}$

d) $\lim_{n\to\infty} \frac{1}{n^2}(1+2+3+\cdots+n)$ $\left\{\frac{1}{2}\right\}$

e) $\lim_{n\to\infty} \frac{n!}{(n+1)!-n!}$ $\{0\}$

f) $\lim_{n\to\infty} \frac{2^n-3^n}{2^n+3^n}$ $\{-1\}$

g) $\lim_{n\to\infty} \frac{3^n+5\cdot 4^n}{3\cdot 4^n-2\cdot 3^n}$ $\left\{\frac{5}{3}\right\}$

5. V R řešte rovnice:

a) $\frac{8}{x+10} = 1 - \frac{3}{x} + \frac{9}{x^2} - \frac{27}{x^3} + \cdots$ $\{-6; 4\}$

b) $\frac{5}{3} = x + 3x^2 + x^3 + 3x^4 + \cdots$ $\left\{-\frac{5}{7}; \frac{1}{2}\right\}$

c) $\log x + \log\sqrt{x} + \log\sqrt[4]{x} + \cdots = 2$ $\{10\}$

d) $2^x + 4^x + 8^x + 16^x + \cdots = 1$ $\{-1\}$

e) $1 + \frac{2}{x} + \frac{4}{x^2} + \frac{8}{x^3} + \cdots = \frac{4x-3}{3x-4}$ $\{6\}$

f) $(2^x)^2 - \frac{32}{3} = 2^x + 2^{x-2} + 2^{x-4} + \cdots$ $\{2\}$

6. Vypočtěte součet nekonečné geometrické řady:

a) $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \frac{81}{256} \ldots$ $\left\{\frac{4}{7}\right\}$

b) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{9} + \frac{1}{8} + \frac{1}{27} + \cdots$ $\left\{\frac{3}{2}\right\}$

c) $\left(\sqrt{3}-\sqrt{2}\right) + \left(\sqrt{3}-\sqrt{2}\right)^2 + \left(\sqrt{3}-\sqrt{2}\right)^3 + \cdots$ $\left\{\frac{\sqrt{6}+\sqrt{2}-2}{4}\right\}$

d) $\left(\sqrt{5}-2\right) + \left(\sqrt{5}-2\right)^2 + \left(\sqrt{5}-2\right)^3 + \cdots$ $\left\{\frac{\sqrt{5}-1}{4}\right\}$

7. Vypočítejte:

a) $A = \frac{1+2+3+\cdots+n}{n+\frac{n}{2}+\frac{n}{4}+\cdots}$ $\left\{A = \frac{n+1}{4}, n \neq 0\right\}$

b) $B = 3 \cdot \sqrt{3} \cdot \sqrt[4]{3} \cdot \sqrt[8]{3} \cdot \ldots$ $\{B = 9\}$