

ЗАДАЧА 11.1. ВАР. А

1) а) $\left(\begin{array}{ccc|c} 2 & 1 & -2 & 7 \\ -1 & 2 & 1 & -1 \\ 2 & 3 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 5 & 0 & 5 \\ 0 & 7 & 3 & -2 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & -5 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array} \right)$ ЗС: $z = -3$
 $y = 1$
 $x - 2y + z + 1 = 2 - 3 + 1 = 0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

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б) ИМ: $\left(\begin{array}{ccc|ccc} 2 & 1 & -2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & -1 & 0 \\ 0 & 5 & 0 & 1 & 2 & 0 \\ 0 & 7 & 3 & 0 & 2 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2/5 & -1/5 & 0 \\ 0 & 1 & 0 & 1/5 & 2/5 & 0 \\ 0 & 0 & 3 & -7/5 & -4/5 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/5 & -7/5 & 1/3 \\ 0 & 1 & 0 & 1/5 & 2/5 & 0 \\ 0 & 0 & 1 & -7/5 & -4/5 & 1/3 \end{array} \right) \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/5 & -7/5 & 1/3 \\ 1/5 & 2/5 & 0 \\ -7/5 & -4/5 & 1/3 \end{pmatrix} \begin{pmatrix} 7 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

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2) $A[0; -3; 5] \vec{u} = (3; 2; 6)$
 $\vec{r} = (1; 2; -3)$

$p: ax + by + cz + d = 0$
 $-18x + 15y + 4z + d = 0$

$p: -18x + 15y + 4z + 25 = 0$
 $18x - 15y - 4z - 25 = 0$

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$\vec{n}_p = \vec{u} \times \vec{r} = \left(\begin{vmatrix} 2 & 6 \\ -3 & 6 \end{vmatrix}, \begin{vmatrix} 3 & 6 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} \right) = (12; 15; 4)$ Асп: $-18 \cdot 0 + 15(-3) + 4 \cdot 5 + d = 0$
 $d = 45 - 20 = 25$

3) $p: \begin{cases} x + 2y - 3z + 6 = 0 \\ 2x - 2y + 4z - 9 = 0 \end{cases} \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & -6 \\ 2 & -2 & 4 & 9 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & -6 \\ 0 & -6 & 10 & 21 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & -6 \\ 0 & 1 & -5/3 & -7/2 \end{array} \right)$ или $z = 3t$
 $y = \frac{5}{3}z - \frac{7}{2} = 5t - \frac{7}{2}$
 $x = -2y + 3z - 6 = -10t + 7 + 9t - 6 = 1 - t$

$p: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -7/2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} t, t \in \mathbb{R}$

Асп: $[1; -7/2; 0]$
 $\vec{n}_p = (-1; 5; 3)$

канониче'ска' р: $\frac{x-1}{-1} = \frac{y+7/2}{5} = \frac{z}{3}$

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4) $k[1; 2; 3] p: \frac{x-5}{1} = \frac{y-4}{-1} = \frac{z-5}{2}$ Асп: $[5; 4; 5] p: x = 5 + t$
 $\vec{n}_p = (1; -1; 2)$ $y = 4 - t$
 $z = 5 + 2t, t \in \mathbb{R}$

$k' = p \cap p: (5+t) - (4-t) + 2(5+2t) - 5 = 0$
 $t = -1$

$k' [4; 5; 3]$

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$p \cap k: \vec{n}_p \cdot \vec{n}_k = 0$
 $p \cap p: \vec{n}_p \cdot \vec{n}_p = (1; -1; 2) \cdot (1; -1; 2) = 6$
 $p: x - y + 2z + d = 0$
 $k \cap p: 1 - 2 + 6 + d = 0 \Rightarrow d = -5$
 $p: x - y + 2z - 5 = 0$

$\sin(30 - 20) = \sin 10 = 0$

5.1: $z = \ln \frac{x}{y}$ $A[4; 5; 24]$

$\frac{\partial z}{\partial x} = \frac{1}{x} \cdot \frac{1}{y} = \frac{1}{xy} = \frac{1}{4 \cdot 5} = \frac{1}{20}$

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$\frac{\partial z}{\partial y} = \frac{1}{x} \cdot x \cdot \left(-\frac{1}{y^2}\right) = -\frac{1}{y^2} = -\frac{1}{25}$

$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{x^2} \quad \frac{\partial^2 z}{\partial x^2}(A) = -\frac{1}{16}$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 0$

$\frac{\partial^2 z}{\partial y^2} = -\left(-\frac{1}{y^2}\right) = \frac{1}{y^2} \quad \frac{\partial^2 z}{\partial y^2}(A) = \frac{1}{25}$

5.2: $z = \sin(3x - 2y)$ $A[0; 0; 24]$

$\frac{\partial z}{\partial x} = \cos(3x - 2y) \cdot 3 \quad \frac{\partial z}{\partial x}(A) = \cos 0 \cdot 3 = 3$

$\frac{\partial z}{\partial y} = \cos(3x - 2y) \cdot (-2) \quad \frac{\partial z}{\partial y}(A) = \cos 0 \cdot (-2) = -2$

$\kappa: z - 0 = 3(x - 0) - 2(y - 0)$

$\kappa: 3x - 2y - z = 0$

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